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# **spherical\_kde Documentation**

***Release 0.1.0***

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**Sep 21, 2020**



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## SPHERICAL\_KDE

### 1.1 spherical\_kde package

#### 1.1.1 Subpackages

**spherical\_kde.tests package**

**Submodules**

**spherical\_kde.tests.test\_distributions module**

```
spherical_kde.tests.test_distributions.random_VonMisesFisher_distribution()  
spherical_kde.tests.test_distributions.random_phi_theta_sigma()  
spherical_kde.tests.test_distributions.test_VonMisesFisher_distribution_mean()  
spherical_kde.tests.test_distributions.test_VonMisesFisher_distribution_normalisation()  
spherical_kde.tests.test_distributions.test_VonMisesFisher_mean()  
spherical_kde.tests.test_distributions.test_VonMisesFisher_standarddeviation()
```

**spherical\_kde.tests.test\_kde module**

```
spherical_kde.tests.test_kde.random_kde(nsamples)  
spherical_kde.tests.test_kde.test_kde_bandwidth_automatic()  
spherical_kde.tests.test_kde.test_kde_correct()  
spherical_kde.tests.test_kde.test_kde_incorrect_lengths()  
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spherical_kde.tests.test_kde.test_kde_lengths()  
spherical_kde.tests.test_kde.test_kde_normalised()  
spherical_kde.tests.test_kde.test_kde_plotting()
```

## spherical\_kde.tests.test\_utils module

```
spherical_kde.tests.test_utils.test_cartesian_from_polar_array()
spherical_kde.tests.test_utils.test_cartesian_from_polar_scalar()
spherical_kde.tests.test_utils.test_decra_from_polar_array()
spherical_kde.tests.test_utils.test_decra_from_polar_scalar()
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spherical_kde.tests.test_utils.test_polar_from_decra_scalar()
spherical_kde.tests.test_utils.test_rotation_matrix()
spherical_kde.tests.test_utils.test_spherical_integrate()
spherical_kde.tests.test_utils.test_spherical_kullback_liebler()
```

## Module contents

### 1.1.2 Submodules

#### 1.1.3 spherical\_kde.distributions module

Module containing the kernel function for the spherical KDE.

For more detail, see: [https://en.wikipedia.org/wiki/Von\\_Mises-Fisher\\_distribution](https://en.wikipedia.org/wiki/Von_Mises-Fisher_distribution)

```
spherical_kde.distributions.VonMisesFisher_distribution(phi, theta, phi0, theta0,
                                                    sigma0)
```

Von-Mises Fisher distribution function.

#### Parameters

**phi, theta** [float or array\_like] Spherical-polar coordinates to evaluate function at.

**phi0, theta0** [float or array-like] Spherical-polar coordinates of the center of the distribution.

**sigma0** [float] Width of the distribution.

#### Returns

**float or array\_like** log-probability of the vonmises fisher distribution.

## Notes

**Wikipedia:** [https://en.wikipedia.org/wiki/Von\\_Mises-Fisher\\_distribution](https://en.wikipedia.org/wiki/Von_Mises-Fisher_distribution)

`spherical_kde.distributions.VonMisesFisher_sample` (*phi0, theta0, sigma0, size=None*)  
Draw a sample from the Von-Mises Fisher distribution.

### Parameters

**phi0, theta0** [float or array-like] Spherical-polar coordinates of the center of the distribution.  
**sigma0** [float] Width of the distribution.  
**size** [int, tuple, array-like] number of samples to draw.

### Returns

**phi, theta** [float or array\_like] Spherical-polar coordinates of sample from distribution.

`spherical_kde.distributions.VonMises_mean` (*phi, theta*)  
Von-Mises sample mean.

### Parameters

**phi, theta** [array-like] Spherical-polar coordinate samples to compute mean from.

### Returns

**float**  
**..math::**  $\sum_i x_i / \|\sum_i x_i\|$

## Notes

**Wikipedia:** [https://en.wikipedia.org/wiki/Von\\_Mises-Fisher\\_distribution#Estimation\\_of\\_parameters](https://en.wikipedia.org/wiki/Von_Mises-Fisher_distribution#Estimation_of_parameters)

`spherical_kde.distributions.VonMises_std` (*phi, theta*)  
Von-Mises sample standard deviation.

### Parameters

**phi, theta** [array-like] Spherical-polar coordinate samples to compute mean from.

### Returns

**solution for** **..math::**  $1/\tanh(x) - 1/x = R$ ,  
where  
**..math::**  $R = \|\sum_i x_i\| / N$

## Notes

**Wikipedia:** [https://en.wikipedia.org/wiki/Von\\_Mises-Fisher\\_distribution#Estimation\\_of\\_parameters](https://en.wikipedia.org/wiki/Von_Mises-Fisher_distribution#Estimation_of_parameters) but re-parameterised for sigma rather than kappa.

## 1.1.4 spherical\_kde.utils module

### Utilities

- General stable functions
- Transforming coordinates
- Computing rotations
- Performing spherical integrals

`spherical_kde.utils.cartesian_from_polar(phi, theta)`

Embedded 3D unit vector from spherical polar coordinates.

#### Parameters

**phi, theta** [float or numpy.array] azimuthal and polar angle in radians.

#### Returns

**nhat** [numpy.array] unit vector(s) in direction (phi, theta).

`spherical_kde.utils.decra_from_polar(phi, theta)`

Convert from ra and dec to spherical polar coordinates.

#### Parameters

**phi, theta** [float or numpy.array] azimuthal and polar angle in radians

#### Returns

**ra, dec** [float or numpy.array] Right ascension and declination in degrees.

`spherical_kde.utils.logsinh(x)`

Compute  $\log(\sinh(x))$ , stably for large  $x$ .

#### Parameters

**x** [float or numpy.array] argument to evaluate at, must be positive

#### Returns

**float or numpy.array**  $\log(\sinh(x))$

`spherical_kde.utils.polar_from_cartesian(x)`

Embedded 3D unit vector from spherical polar coordinates.

#### Parameters

**x** [array\_like] cartesian coordinates

#### Returns

**phi, theta** [float or numpy.array] azimuthal and polar angle in radians.

`spherical_kde.utils.polar_from_decra(ra, dec)`

Convert from spherical polar coordinates to ra and dec.

#### Parameters

**ra, dec** [float or numpy.array] Right ascension and declination in degrees.

#### Returns

**phi, theta** [float or numpy.array] Spherical polar coordinates in radians

`spherical_kde.utils.rotation_matrix(a, b)`

The rotation matrix that takes  $a$  onto  $b$ .



**Parameters**

**a, b** [numpy.array] Three dimensional vectors defining the rotation matrix

**Returns**

**M** [numpy.array] Three by three rotation matrix

**Notes**

**StackExchange post:** <https://math.stackexchange.com/questions/180418/calculate-rotation-matrix-to-align-vector-a-to-vector-b>

`spherical_kde.utils.spherical_integrate(f, log=False)`

Integrate an area density function over the sphere.

**Parameters**

**f** [callable] function to integrate (phi, theta) -> float

**log** [bool] Should the function be exponentiated?

**Returns**

**float** Spherical area integral

$$\int_0^{2\pi} d\phi \int_0^\pi d\theta f(\phi, \theta) \sin(\theta)$$

`spherical_kde.utils.spherical_kullback_liebler(logp, logq)`

Compute the spherical Kullback-Liebler divergence.

**Parameters**

**logp, logq** [callable] log-probability distributions (phi, theta) -> float

**Returns**

**float** Kullback-Liebler divergence

$$\int P(x) \log \frac{P(x)}{Q(x)} dx$$

**Notes**

**Wikipedia post:** [https://en.wikipedia.org/wiki/Kullback-Leibler\\_divergence](https://en.wikipedia.org/wiki/Kullback-Leibler_divergence)

## 1.1.5 Module contents

The spherical kernel density estimator class.

**class** `spherical_kde.SphericalKDE(phi_samples, theta_samples, weights=None, bandwidth=None, density=100)`

Bases: `object`

Spherical kernel density estimator

**Parameters**

**phi\_samples, theta\_samples** [array\_like] spherical-polar samples to construct the kde

**weights** [array\_like] Sample weighting default `[1] * len(phi_samples)`

**bandwidth** [float] bandwidth of the KDE. Increasing bandwidth increases smoothness

**density** [int] number of grid points in theta and phi to draw contours.

#### Attributes

**phi, theta** [numpy.array] spherical polar samples

**weights** [numpy.array] Sample weighting (normalised to sum to 1).

**bandwidth** [float] Bandwidth of the kde. defaults to rule-of-thumb estimator [https://en.wikipedia.org/wiki/Kernel\\_density\\_estimation](https://en.wikipedia.org/wiki/Kernel_density_estimation) Set to None to use this value

**density** [int] number of grid points in theta and phi to draw contours.

**palefactor** [float] getdist-style colouration factor of sigma-contours.

#### Methods

<code>__call__(phi, theta)</code>	Log-probability density estimate
<code>plot(ax[, colour])</code>	Plot the KDE on an axis.
<code>plot_samples(ax[, nsamples])</code>	Plot equally weighted samples on an axis.

#### property bandwidth

**plot** (*ax*, *colour*='g', *\*\*kwargs*)  
Plot the KDE on an axis.

#### Parameters

**ax** [matplotlib.axes.Axes] matplotlib axis to plot on. This must be constructed with a *cartopy.crs.projection*:

```
>>> import cartopy
>>> import matplotlib.pyplot as plt
>>> fig = plt.subplots()
>>> ax = fig.add_subplot(111, projection=cartopy.crs.Mollweide())
```

**color** Colour to plot the contours. *arg* can be an *RGB* or *RGBA* sequence or a string in any of several forms:

- 1) a letter from the set 'rgbcmykw'
- 2) a hex color string, like '#00FFFF'
- 3) a standard name, like 'aqua'
- 4) a string representation of a float, like '0.4',

This is passed into *matplotlib.colors.colorConverter.to\_rgb*

**plot\_samples** (*ax*, *nsamples*=None, *\*\*kwargs*)  
Plot equally weighted samples on an axis.

#### Parameters

**ax** [matplotlib.axes.Axes] matplotlib axis to plot on. This must be constructed with a *cartopy.crs.projection*:

```
>>> import cartopy
>>> import matplotlib.pyplot as plt
>>> fig = plt.subplots()
>>> ax = fig.add_subplot(111, projection=cartopy.crs.Mollweide())
```

**nsamples** [int] Approximate number of samples to plot. Can only thin down to this number, not bulk up



## INDICES AND TABLES

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